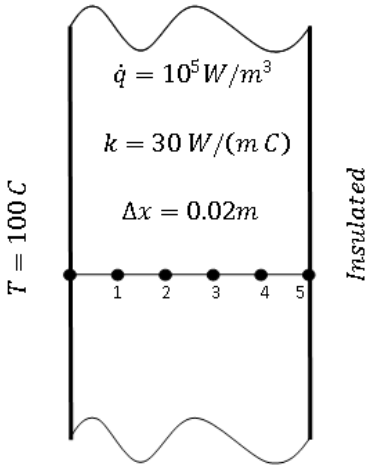
Sheet five

A wall is subjected to boundary conditions as shown in the following problems where the wall is large relative to its thickness

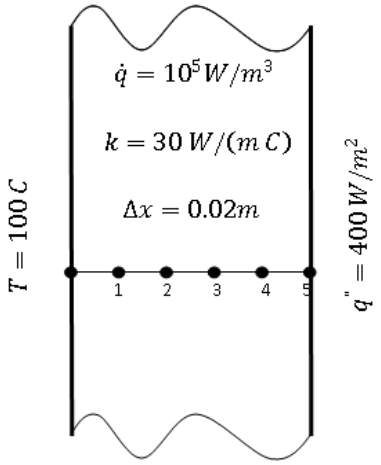
Determine

a) The nodal finite-difference equations
b) The nodal temperature using Gauss-Seidel Iteration with maximum relative error of ε and initial value of T_i as shown in following table

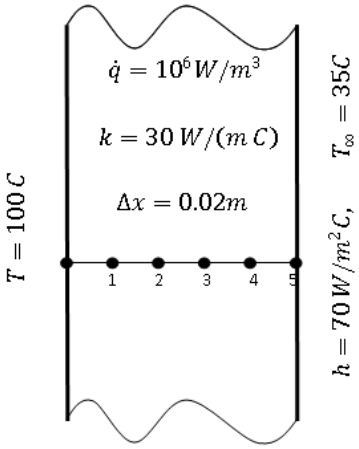
Problem	1	2	3	4	5	6
ε	0.003	0.004	0.006	0.003	0.006	0.006
T_i	110	110	200	180	150	40



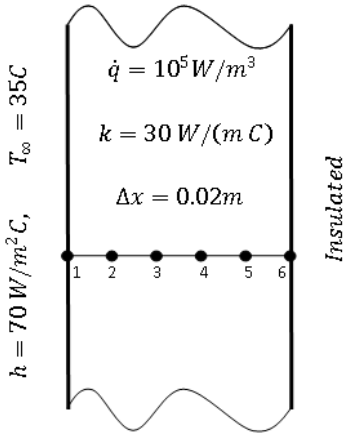
Problem 1



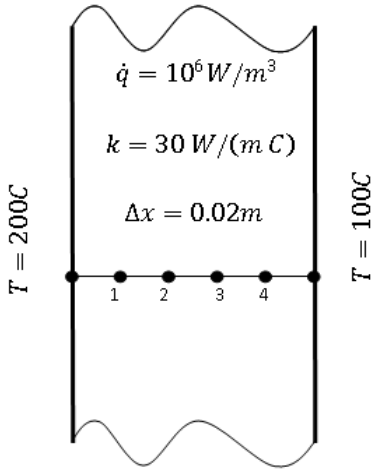
Problem 2



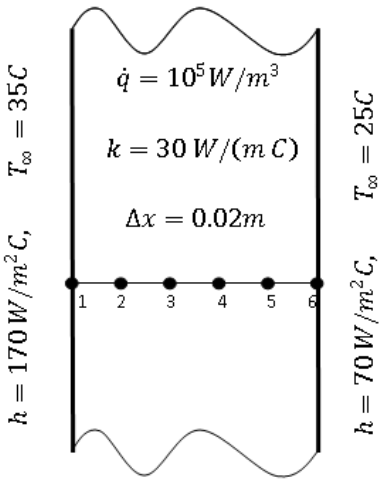
Problem 3



Problem 4



Problem 5



Problem 6

1 - السؤال الاول والثالث محلولين حل نموذجى (مرفق مع الشيت)
2 - السؤال الثانى والسادس سيتم شرحهم فى السكشن
3 - السؤال الرابع والخامس سيحلهم الطالب ويقدمهم فى تقرير منظم فى الموعد الذى سيحدده المعيد
4 - فى حالة تقديم التقرير بعد الموعد المحدد فلن يقبل منه مهما كانت الاعذار ولن توضع له درجة

Solution of problems 1 and 3

1)

$a)\ The\ nodal\ finite\ -\ difference\ equations$

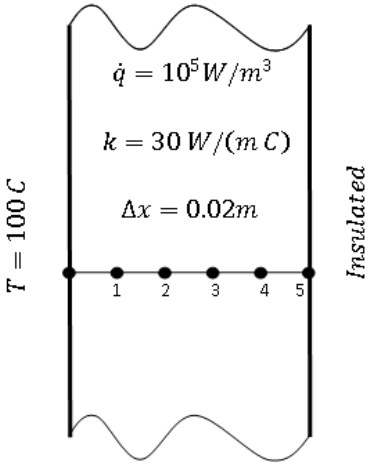
$Node\ 1$

$$\dot{q}\Delta x + k\frac{100 - T_1}{\Delta x} + k\frac{T_2 - T_1}{\Delta x} = 0 \times \frac{\Delta x}{k}$$

$$\frac{\dot{q}\Delta x^2}{k} + 100 - T_1 + T_2 - T_1 = 0$$

$$\frac{100000 \times 0.02^2}{30} + 100 + T_2 - (1+1)T_1 = 0$$

$$T_1 = 50.6667 + 0.5\,T_2$$



Problem 1

$Node\ 2$

$$\dot{q}\Delta x + k\frac{T_1 - T_2}{\Delta x} + k\frac{T_3 - T_2}{\Delta x} = 0 \times \frac{\Delta x}{k}$$

$$\frac{\dot{q}\Delta x^2}{k} + T_1 - T_2 + T_3 - T_2 = 0$$

$$\frac{100000 \times 0.02^2}{30} + T_1 + T_3 - (1+1)T_2 = 0$$

$$T_2 = 0.666667 + 0.5\,T_1 + 0.5\,T_3$$

$Node\ 3$

$$\dot{q}\Delta x + k\frac{T_2 - T_3}{\Delta x} + k\frac{T_4 - T_3}{\Delta x} = 0 \times \frac{\Delta x}{k}$$

$$\frac{\dot{q}\Delta x^2}{k} + T_2 - T_3 + T_4 - T_3 = 0$$

$$\frac{100000 \times 0.02^2}{30} + T_2 + T_4 - (1+1)T_3 = 0$$

$$T_3 = 0.666667 + 0.5\,T_2 + 0.5\,T_4$$

$Node\ 4$

$$\dot{q}\Delta x + k\frac{T_3 - T_4}{\Delta x} + k\frac{T_5 - T_4}{\Delta x} = 0 \times \frac{\Delta x}{k}$$

$$\frac{\dot{q}\Delta x^2}{k} + T_3 - T_4 + T_5 - T_4 = 0$$

$$\frac{100000 \times 0.02^2}{30} + T_3 + T_5 - (1+1)T_4 = 0$$

$$T_4 = 0.666667 + 0.5\,T_3 + 0.5\,T_5$$

$Node\ 5$

$$\frac{\dot{q}\Delta x}{2} + k\frac{T_4 - T_5}{\Delta x} = 0 \quad \times \frac{\Delta x}{k}$$

$$\frac{\dot{q}\Delta x^2}{2k} + T_4 - T_5 = 0$$

$$\frac{100000 \times 0.02^2}{2 \times 30} + T_4 - (1)T_5 = 0$$
$$T_5 = 0.666667 + T_4$$

b) The nodal temperature using Gauss-Seidel Iteration with relative error of 0.003 and initial value of 110

$$T_1 = 50.6667 + 0.5\,T_2$$
$$T_2 = 0.666667 + 0.5\,T_1 + 0.5\,T_3$$
$$T_3 = 0.666667 + 0.5\,T_2 + 0.5\,T_4$$
$$T_4 = 0.666667 + 0.5\,T_3 + 0.5\,T_5$$
$$T_5 = 0.666667 + T_4$$

$$error = \left|\frac{t_{new} - t_{old}}{t_{old}}\right|$$

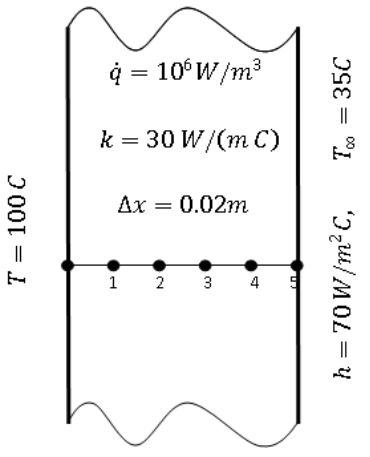
	1	error	2	error	3	error	4	error	5	error	6	error
110	105.667	0.0394	104.917	0.0071	104.708	0.0020	104.682	0.0002	104.757	0.0007	104.857	0.0010
110	108.5		108.083		108.031	0.0005	108.18	0.0014	108.382	0.0019	108.587	0.0019
110	109.917		110.021		110.344	0.0029	110.673	0.0030	110.984	0.0028	111.27	0.0026
110	110.625		111.323		111.833	0.0046	112.253	0.0038	112.619	0.0033	112.944	0.0029
110	111.292		111.99		112.5		112.92		113.285		113.611	0.0029

3)

a) The nodal finite − difference equations

Node 1

$$\dot{q}\Delta x + k\frac{100 - T_1}{\Delta x} + k\frac{T_2 - T_1}{\Delta x} = 0 \times \frac{\Delta x}{k}$$
$$\frac{\dot{q}\Delta x^2}{k} + 100 - T_1 + T_2 - T_1 = 0$$
$$\frac{1000000 \times 0.02^2}{30} + 100 + T_2 - (1+1)T_1 = 0$$
$$T_1 = 56.6667 + 0.5\,T_2$$



Problem 3

Node 2

$$\dot{q}\Delta x + k\frac{T_1 - T_2}{\Delta x} + k\frac{T_3 - T_2}{\Delta x} = 0 \times \frac{\Delta x}{k}$$
$$\frac{\dot{q}\Delta x^2}{k} + T_1 - T_2 + T_3 - T_2 = 0$$
$$\frac{1000000 \times 0.02^2}{30} + T_1 + T_3 - (1+1)T_2 = 0$$
$$T_2 = 6.66667 + 0.5\,T_1 + 0.5\,T_3$$

Node 3

$$\dot{q}\Delta x + k\frac{T_2 - T_3}{\Delta x} + k\frac{T_4 - T_3}{\Delta x} = 0 \times \frac{\Delta x}{k}$$

$$\frac{\dot{q}\Delta x^2}{k} + T_2 - T_3 + T_4 - T_3 = 0$$
$$\frac{1000000 \times 0.02^2}{30} + T_2 + T_4 - (1+1)T_3 = 0$$
$$T_3 = 6.66667 + 0.5\,T_2 + 0.5\,T_4$$

Node 4

$$\dot{q}\Delta x + k\frac{T_3 - T_4}{\Delta x} + k\frac{T_5 - T_4}{\Delta x} = 0 \times \frac{\Delta x}{k}$$
$$\frac{\dot{q}\Delta x^2}{k} + T_3 - T_4 + T_5 - T_4 = 0$$
$$\frac{1000000 \times 0.02^2}{30} + T_3 + T_5 - (1+1)T_4 = 0$$
$$T_4 = 6.66667 + 0.5\,T_3 + 0.5\,T_5$$

Node 5

$$\frac{\dot{q}\Delta x}{2} + h\,(T_\infty - T_5) + k\frac{T_4 - T_5}{\Delta x} = 0 \quad \times \frac{\Delta x}{k}$$
$$\frac{\dot{q}\Delta x^2}{2k} + \frac{\Delta x h T_\infty}{k} - \frac{\Delta x h}{k}T_5 + T_4 - T_5 = 0$$
$$\frac{1000000 \times 0.02^2}{2 \times 30} + \frac{0.02 \times 70 \times 35}{30} + T_4 - \left(\frac{0.02 \times 70}{30} + 1\right)T_5 = 0$$
$$T_5 = 7.92994 + 0.955414\,T_4$$

b) The nodal temperature using Gauss-Seidel Iteration with relative error of 0.006 and initial value of 200

$$T_1 = 56.6667 + 0.5\,T_2$$
$$T_2 = 6.66667 + 0.5\,T_1 + 0.5\,T_3$$
$$T_3 = 6.66667 + 0.5\,T_2 + 0.5\,T_4$$
$$T_4 = 6.66667 + 0.5\,T_3 + 0.5\,T_5$$
$$T_5 = 7.92994 + 0.955414\,T_4$$
$$error = \left|\frac{t_{new} - t_{old}}{t_{old}}\right|$$

0	1	*error*	2	*error*	3	*error*	4	*error*	5	*error*	6	*error*
200	156.667	0.2167	149.167	0.0479	147.083	0.0140	146.823	0.0018	147.069	0.0017	147.45	0.0026
200	185		180.833		180.313		180.805	0.0027	181.567	0.0042	182.354	0.0043
200	199.167		200.208		201.454		202.732	0.0063	203.925	0.0059	205.004	0.0053
200	206.25		209.263		211.325		212.949		214.321	0.0064	215.517	0.0056
200	204.984		207.863		209.833		211.385		212.696		213.838	0.0054